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CONTACT INTERACTION BETWEEN A CYLINDRICAL PANEL AND A HALF-PLANE

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FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

Let an external load be applied to a shell and directed parallel to the edge of a semi-infinite plate, as is shown in Fig. 1. The bodies are joined in sections whose width h_0 is small compared to the length $2l$, so that the contact domain can be considered a straight-line segment $\{x \in [-l, l], y = 0\}$. We take the density of the tangential contact forces $\tau(x)$ as the principal unknown. We set the normal component equal to zero. This is justified physically by the fact that the bending stiffness of a thin-walled panel is considerably less than the tension—compression stiffness. An analogous simplification is used in [1] in analyzing the contact interaction between shells and is, in mathematical respects, that we have one singular integral equation in place of a system of two. To obtain it we take the equality of the strains $(u_0)'_x$ in the plate and u'_x in the shell as the contact condition. Using the Green's function from [1, 2], we have

$$\int_{-l}^{l} \tau(\xi)\, \Phi(x-\xi)\, d\xi = f(x), \qquad (1)$$

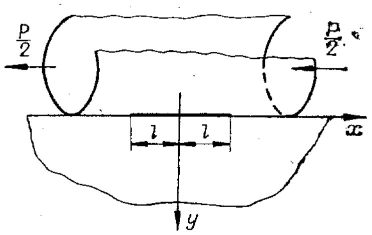

Fig. 1

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where

$$\Phi(t) = C[t^{-1} + S(t)];$$
$$S(t) = (Cl)^{-1} \sum_{m=1}^{\infty} \varepsilon^{2m} (tl^{-1})^{2m-1} \left(c_{1m} + c_{2m} \ln \frac{|t|}{l}\right); \tag{2}$$
$$C = -(1+\nu)(3-\nu)(4\pi Eh)^{-1} - 2(\pi E_0 h_0)^{-1};\ \varepsilon = 0{,}5bl;\ b^4 = 12(1-\nu^2)h^{-2}R^{-2};$$

E, ν, h, and R are the shell elastic modulus, Poisson ratio, thickness, and radius; E_0 and h_0 are the plate elastic modulus and thickness; and f(x) is the panel strain in the contact zone due to external loads.

The expressions c_{sm} (s = 1, 2) dependent on m and ln ε are not presented. They can be obtained from [2]. The decrease in c_{sm} assures convergence of the series (2) and two of its derivatives on the whole number axis.

To seek the density $\tau(x)$ we convert (1) by the method of equivalent regularization into a Fredholm integration equation of the second kind

$$Q(x) = Q_0(x) + L[Q(x)], \tag{3}$$
$$Q(x) = \tau(x)(l^2 - x^2)^{1/2};$$

where

$$Q_0 = P\pi^{-1} + (\pi^2 C)^{-1} \int_{-l}^{l} (l^2 - t^2)^{1/2} f(t) (t-x)^{-1}\, dt;$$

$$P = \int_{-l}^{l} \tau(x)\, dx; \quad L = -\pi^{-2} \int_{-l}^{l} Q(\xi) (l^2 - \xi^2)^{-1/2} F(\xi)\, d\xi;$$

$$F(\xi) = \int_{-l}^{l} (l^2 - t^2)^{1/2} (t-x)^{-1} S(t-\xi)\, dt.$$

The order of integration in the operator L is interchanged. This is possible by virtue of the continuity of the function $S(t-\xi)$ [1].

In the half-interval $\varepsilon \in [0; \varepsilon_1[$ we determine the function $\tau(x)$ approximately from (3) by successive approximations. By using the tabulated integrals in [5], we find, analogously to [3, 4],

$$\varepsilon_1 = (2D_1)^{-1} \left[(D_2^2 + 4D_1)^{1/2} - D_2 \right],$$
$$D_1 = \pi^2 c_1 (1 - 2\nu + 5\nu^2) [128(1+\nu)]^{-1}, \quad D_2 = \pi^2 c_1 (3 - 2\nu + 3\nu^2) [32\sqrt{2}(1+\nu)]^{-1},$$
$$c_1 = \left\{ \frac{1}{8} \pi (3-\nu) + \pi Eh [E_0 h_0 (1+\nu)]^{-1} \right\}^{-1}.$$

Here for ν = 0.3 the value ε_1 = 4.99 corresponds to identical thicknesses and materials of the bodies making contact.

Let us examine specific problems further.

UNIFORM PANEL TENSION

Let the shell be stretched so that its strain in the longitudinal direction is constant and equal to Δ. Let us clarify the distribution of the contact forces and stresses in the juncture zone with the half plane. Since the forces are self-equilibrated, we construct the solution of (3) under the conditions that $\int_{-l}^{l} \tau(x)\, dx = 0$, $f(x) = \Delta$. To the accuracy of components on the order of ε^6, we find by successive approximations

$$\tau(x) = (\pi C)^{-1} (l^2 - x^2)^{-\frac{1}{2}} \left[\sum_{k=0}^{2} T_{2k+1} x^{2k+1} + O(\varepsilon^8) \right],$$

TABLE 1

$10xl^{-1}$	Values of $T(x)$				
	ε=0	ε=1,5	ε=2,5	ε=3,5	ε=4,5
0	318	308	289	261	225
1	320	309	291	263	227
2	324	314	297	271	236
3	333	324	308	284	253
4	347	339	326	305	278
5	367	361	351	334	313
6	398	394	390	378	365
7	445	445	444	443	442
8	530	535	543	556	574
9	730	745	771	810	863

where

$$T_1 = -\omega\left[C - 0.5\varepsilon^2 c_{11} + \varepsilon^4\left(0.25c_{11}^2 C^{-1} + 0.375c_{12} - c_{22}\frac{6\chi - 13}{16}\right) + \frac{1}{2}\varepsilon^6\left(\frac{15}{4}c_{13} + \frac{3}{8}\gamma - \frac{1}{4}c_{11}^3 C^{-2} - \frac{6\chi+1}{16}\beta\right)\right];$$

$$T_3 = -\omega\varepsilon^4 l^{-2}\left[c_{22}\left(\frac{3}{2}\chi - \frac{15}{8}\right) - \frac{3}{2}c_{12} + \frac{1}{2}\varepsilon^2\left(\frac{3}{2}\gamma + \frac{15 - 12\chi}{8}\beta - 5c_{13}\right)\right];$$

$$T_5 = \omega\varepsilon^4 l^{-4}\left[\frac{1}{4}c_{22} - \frac{1}{2}\varepsilon^2\left(\frac{1}{4}\beta - 5c_{13}\right)\right];\quad \omega = \Delta C^{-1};\quad \chi = \ln 2;\quad \beta = c_{11}c_{22}C^{-1};$$

$$\gamma = c_{11}c_{12}C^{-1}.$$

PANEL SHEAR RELATIVE TO THE HALF PLANE

Let the external forces of magnitude 0.5 be directed to one side (opposite to the Ox axis) and applied at the points ($y = 0$, $|x| = l_1 >> l$). Then we can set $f(x) = 0$ approximately. The solution of (3) satisfying the equilibrium condition

$$\int_{-l}^{l} \tau(x)\,dx = P$$

is constructed analogously to the preceding solution. To the accuracy of components of the order ε^6 it has the form

$$\tau(x) = P(\pi C)^{-1}(l^2 - x^2)^{-\frac{1}{2}}\left[\sum_{k=0}^{3} T_{2k}x^{2k} + O(\varepsilon^8)\right],$$

where

$$T_0 = C - \frac{1}{2}\varepsilon^2 c_{11} - \varepsilon^4\left[\frac{7}{8}c_{12} + c_{22}\left(\frac{41}{48} - \frac{7}{8}\chi\right)\right] - \varepsilon^6\left(\frac{67 - 30\chi}{160}\beta + \frac{13}{8}c_{13}\right);$$

$$T_2 = \varepsilon^2 l^{-2}\left[c_{11} - \varepsilon^2\left(c_{22}\chi - \frac{1}{3}c_{22} - c_{12}\right) - \varepsilon^4\left(\frac{3}{4}c_{13} + \frac{30\chi - 53}{80}\beta\right)\right];$$

$$T_4 = -\varepsilon^4 l^{-4}\left[c_{22}\left(\chi - \frac{11}{6}\right) - c_{12} - \varepsilon^2\left(\frac{11}{40}\beta + \frac{9}{2}c_{13}\right)\right];$$

$$T_6 = \varepsilon^6 l^{-6}(c_{13} - 0.05\beta).$$

NATURE OF PANEL STRESS STATE SINGULARITIES OUTSIDE THE REINFORCEMENT ZONE

An analysis of the stresses σ_x and σ_y shows that they are bounded in a segment of panel contact with the half plane. However, in connection with the fact that the contact forces are infinite at the points ($|x| = l$, $y = 0$), the stresses have a singularity of order $^1/_2$ outside the reinforcement zone. Thus, by using the two-dimensional Fourier transform [6], we obtain the following asymptotic formulas to calculate the principal values of the stresses outside the contact segment $\{y = 0,\ |x| = l + |r|\,|_{r\to 0}\}$:

For uniform panel tension

$$\sigma_\alpha = MA_\alpha \sqrt{l}(C\sqrt{2|r|})^{-1}; \qquad (4)$$

for shear relative to the half plane

$$\sigma_\alpha = PNA_\alpha \operatorname{sgn}(x)(\sqrt{2l|r|})^{-1}. \qquad (5)$$

Here,

$$\alpha \in \{x;\ y\};\ A_x = -(3+\nu)(4\pi h)^{-1};\ A_y = (1-\nu)(4\pi h)^{-1};$$
$$M = \Delta[1 - \varepsilon^2 c_{11}(2C)^{-1}] + O(\varepsilon^4);\ N = [1 + \varepsilon^2 c_{11}(2C)^{-1}] + O(\varepsilon^4);$$
$$c_{11} = -(32Eh)^{-1}(1 - 2\nu + 5\nu^2).$$

Since σ_x and σ_y are infinite at the ends of the contact segment, the panel strength can be estimated by brittle fracture criteria [7]. The stress intensity factors are calculated elementarily here by using (4) and (5).

Let us note that a considerable concentration of contact forces and stresses on the edges of the domains joining the plate and shell is detected numerically in [8]. However, the nature of the singularity of these quantities remains unknown.

By analyzing (4) and (5), it can be seen that, under uniform panel tension, the stress intensity factors are directly proportional to the square root of the length of the reinforcement segment. An analogous dependence holds in the theory of rupture [7], where the crack extent plays the part of the length. The intensity factors are inversely proportional to l for shell shear relative to the half plane. Hence, by elongating the reinforcement zone in the second case, the level of the stress state can be lowered.

RESULTS OF THE COMPUTATION

Presented in Table 1 are dimensionless values of the contact force $T(x) = 10^3\tau(x)lP^{-1}$ calculated under the assumption that the thickness and material of the bodies making contact are identical, and $\nu = 0.3$. An increase in ε results in elongation of the contact zone. Here the tangential forces in the middle part of the zone diminish and increase at the edges. The value $\varepsilon = 0$ corresponds to contact between the half plane and the plate. There are no qualitative differences in the contact force distributions for the plate and shell, i.e., the curvature of the middle surface of a thin-walled body induces no changes in the order of the singularities. Hence, the rate of convergence of the asymptotic method is sufficiently high when conserving the inequality ($\varepsilon < \varepsilon_1$), where the solution for the plate appears as the zeroth approximation.

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